Review

Sept. 2, 2014

Section 1.1

Exercises:

1) b. proposition

c. $2+3=5$ proposition

e. $x+3=11$ not proposition

3) a. (Mei has a MP3 player.) $= a$

$\neg a =$ Mei does not have a MP3 player

c. $(2+1=3) = a$

$\neg a = 2+1 \neq 3$

d. $\neg$(The summer in Miami is sunny & hot.)

= The summer in Miami is not sunny or not hot

Note: De Morgan's Law

12) a. p: You have a flu.

q: You miss the final exam.

r: You pass the course.

$p \rightarrow q \equiv$ "If you have the flu, then you miss the final exam."

$\neg q \leftrightarrow r \equiv$ "You do not miss the final exam if and only if you pass the course."

15) p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.
See book for sentences that are in need of translation.

a) $r \wedge \neg p$

b) $\neg p \wedge q \wedge r$

c) $r \rightarrow (q \leftrightarrow \neg p)$ Note!: Precedence is ignored because of sentence structure.

27) a) If it snows today, I will ski tomorrow.

Converse: If I will ski tomorrow, then it snows today.

Contrapositive: If I will not ski tomorrow, then it does not snow today.

Inverse: If it does not snow today, I will not ski tomorrow.

b) I come to class whenever there is a quiz.

Con: If I come to class then there is a quiz.

Contr: If I don't go to class, then there won't be a quiz.

Inv: If there is no quiz, then I will not go to class.

c) A positive integer is a prime only if it has no divisors other than one and itself.

Con: If it has divisors other than one and itself, then (the rest)

Contr: If it does not have divisors other than one and itself, then not (the rest)

Inv: If it is not a positive integer, then it does not have divisors other than one and itself.

33) a) $(p \vee q) \rightarrow (p \oplus q)$ Truth table:

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

e) $\underbrace{(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)}_{A}$

p	q	r	$\neg p$	$\neg r$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg r)$	A
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F

Reminder: $\oplus$ works opposite to $\leftrightarrow$

Ex:

p	q	$p \leftrightarrow q$		$p \oplus q$
T	T	T	$\neq$	F
T	F	F	$\neq$	T
F	T	F	$\neq$	T
F	F	T	$\neq$	F

Section 1.3

7) a) Jan is rich and happy.
b) Carlos will bicycle or run tomorrow.
c) Mei walks or takes the bus to class.
d) Ibrahim is smart and hard-working
*) If it is rainy, I'll stay home

$\neg a$ = Jan is not rich or not happy.
$\neg b$ = Carlos will not bicycle and will not run tomorrow.
$\neg c$ = Mei does not walk and does not take the bus to class.
$\neg d$ = Ibrahim is not smart or not hard-working.
$\neg *$ = I will not stay home and it is rainy.

Explanation:

$$\underbrace{\text{If it is rainy}}_{p}, \underbrace{\text{I'll stay home.}}_{q}$$

$p \to q \equiv q \vee \neg p$ by "Star" Law

$\neg(p \to q) \equiv \neg(q \vee \neg p) \equiv \neg q \wedge p$ by De Morgan's Law

9) a) $p \wedge q \rightarrow p$; Is it a tautology?

p	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$\boxed{\text{Yes!}}$

10) a) $(\neg p \wedge (p \vee q)) \rightarrow q$; Is it a tautology?

~~p~~	~~q~~	~~$\neg p$~~	$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$\boxed{\text{Yes!}}$

21) $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$; Are they equivalent?
w/ truth table:

p	q	$\neg p$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$		$\neg p \leftrightarrow q$
T	T	F	T	F	=	F
T	F	F	F	T	=	T
F	T	T	F	T	=	T
F	F	T	T	F	=	F

$\boxed{\text{Equivalent!}}$

1.3 / Q21

w/ formula:

$\neg(p \leftrightarrow q) \equiv \neg((p \to q) \wedge (q \to p))$ by definition of biconditional connective

$\equiv \neg(p \to q) \vee \neg(q \to p)$ by De Morgan's Law

$\equiv \neg(q \vee \neg p) \vee \neg(p \vee \neg q)$ by "Star" Laws

$\equiv (\neg q \wedge p) \vee (\neg p \wedge q)$ by De Morgan's Laws

$\equiv [(\neg q \wedge p) \vee \neg p] \wedge [(\neg q \wedge p) \vee q]$ by Distributive Law

$\equiv [(\neg p \vee \neg q) \wedge \underbrace{(\neg p \vee p)}_{T}] \wedge [\underbrace{(q \vee \neg q)}_{T} \wedge (q \vee p)]$ by Distributive

$\equiv (\neg p \vee \neg q) \wedge (q \vee p)$ by Identity Law

$\equiv (q \to \neg p) \wedge (\neg p \to q)$ by "Star" Law

$\equiv q \leftrightarrow \neg p$ by definition of biconditional

$\equiv \neg p \leftrightarrow q$ Yay!

26) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

$\neg p \to (q \to r) \equiv (q \to r) \vee p \equiv (r \vee \neg q) \vee p \equiv (p \vee r) \vee \neg q$

$\equiv q \to (p \vee r)$