Sept. 9, 2014

Extra Session Material.

Sec. 1.4

6. $N(x)$ = "x has visited North Dakota" where the domain is students in your school.

e) $\neg \forall x\, N(x)$

Not all students have visited North Dakota.

c) $\neg \exists x\, N(x)$

No student has visited North Dakota.

9. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers and logical connectives. Domain - students in your school.

* See book for sentences *

a) $\exists x\, (P(x) \wedge Q(x))$

b) $\exists x\, (P(x) \wedge \neg Q(x))$

d) $\forall x\, \neg(P(x) \vee Q(x))$ or $\neg \exists x\, (P(x) \vee Q(x))$

29. Express each of these statements using logical operators, predicates, and quantifiers.

a) Some propositions are tautologies.
Solution: Assume the domain are all propositions.
$T(x)$ = x is a tautology

$\exists x\, T(x)$

12. Let $Q(x)$ be the statement "$x+1 > 2x$". If the domain consists of all integers, what are these truth values?

a) $Q(0) = 0+1 > 2(0) = 1 > 0$ True
f) $\exists x \neg Q(x)$ = "Some x does not make $x+1 > 2x$ true"
True.

14. Determine the truth value of each of these statements if the domain consists of all real numbers.
a) $\exists x (x^3 = -1)$ True
c) $\forall x ((-x)^2 = x^2)$ True

18. Suppose that the domain of the propositional function $P(x)$ consists of the integers $-2, -1, 0, 1$ and 2. Write out of these propositions using justifications,

conjunctions, and negations.

c) $\exists_x \neg P(x)$ $\quad U = \{-2, -1, 0, 1, 2\}$

$\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

or

$\neg (P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

20. Suppose the domain of the propositional function $P(x)$ consists of $-5, -3, -1, 1, 3$ and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

c) $\forall x ((x \neq 1) \rightarrow P(x))$

$(-5 \neq 1 \rightarrow P(-5)) \wedge (-3 \neq 1 \rightarrow P(-3)) \wedge (-1 \neq 1 \rightarrow P(-3))$
$\wedge (1 \neq 1 \rightarrow P(1)) \wedge (3 \neq 1 \rightarrow P(3)) \wedge (5 \neq 1 \rightarrow P(5))$

52. Look for instructions in book
a) False b) true d) false

22. For each of these statements find a domain for which the statement is true and one for which it is false.

b) There is someone older than 21 years of age.
Truth domain - everyone in the world
False domain - people in Kindergarden

33. *See book for instructions*

a) Some old dogs can learn new tricks.

domain = dogs

$\neg \exists x$ (x is old $\rightarrow$ x can learn new tricks)

$\forall x \neg$ (x is old $\rightarrow$ x can learn new tricks)

$\forall x$ (x cannot learn new tricks $\wedge$ x is old)

43. Determine wether $\forall x (P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

$$\forall x (P(x) \rightarrow Q(x)) \stackrel{?}{\equiv} \forall x P(x) \rightarrow \forall x Q(x)$$

Case: $Q(x) \equiv$ False $\qquad P(x)$: sometimes true and sometimes false

$\forall x\, Q(x) \equiv$ False

$\underbrace{\forall x (P(x) \rightarrow Q(x))}_{F}$ is always false.

$\underbrace{\forall x P(x)}_{T \text{ or } F} \rightarrow \underbrace{\forall x Q(x)}_{F}$ could be true or false

Section 1.5

30. Rewrite each of these statements so that the negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

d) $\neg \exists y \, (\exists x \, R(x,y) \vee \forall x \, S(x,y))$
$\equiv \forall y \, \neg (\exists x \, R(x,y) \vee \forall x \, S(x,y))$
$\equiv \forall y \, (\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))$

35. Find a common domain for the variables x, y, z and w for which the statement:

$$\forall x \, \forall y \, \forall z \, \exists z \, ((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$$

is true and another common domain for these variables for which it is false.

Ⓣ Truth domain: all real numbers
Ⓕ False domain: $\{1, 2, 3\}$

Ⓣ $\forall x \, \forall y \, \forall z \, \exists w \, ((10 \neq 1) \wedge (10 \neq 20) \wedge (10 \neq 300))$ True
Ⓕ $\forall x \, \forall y \, \forall z \, \exists w \, ((3 \neq 1) \wedge (3 \neq 2) \wedge (3 \neq 3))$ False

or 1 or 2

7. Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

d) $\forall x \forall z \exists y ((x \neq z) \rightarrow \neg(T(x, y) \wedge T(z, y)))$

For any two students there exists a cuisine that one student likes and the other dislikes.