Assignment #1 SOLUTION ICOM 4075 FALL 2014

1.1.2 (a) NO (b) NO (c) YES (d) NO (e) YES (f) NO

1.1.4 (a) Jennifer and Teja are not friends

(b) A baker's dozen doesn't have 13 items

(c) Abby sent 100 or less text messages every day

(d) 121 is not a perfect square

1.1.12 (b) You pass the course if and only if you take the final.

(e) You fail the course when you have the flu or miss the final

(f) You either have the flu and miss the final or take the final and pass the course.

1.1.15 (d) $\neg q \wedge \neg p \wedge r$ (e) $q \to \neg p \wedge \neg r$ (f) $p \wedge r \to \neg q$

1.1.28

(a) If $\underbrace{\text{it snows tonight}}_{p = \text{hypothesis}}$, then $\underbrace{\text{I will stay at home}}_{q = \text{conclusion}}$

Converse $(q \to p)$

If I stay home then it snows tonight

Contrapositive $(\neg q \to \neg p)$

If I don't stay home ~~tonight~~ then it does snow tonight.

Inverse: $(\neg p \to \neg q)$

If it doesn't snow tonight, I will stay home

1.1.28 Continued

(b) $\underbrace{\text{I go to the beach}}_{q = \text{conclusion}}$ whenever $\underbrace{\text{it is a sunny summer day}}_{p = \text{hypothesis}}$

Converse $(q \rightarrow p)$

It is a sunny summer day whenever I go to the beach.

Contrapositive $(\neg q \rightarrow \neg p)$

If I don't go to the beach then it is not a sunny summer day.

Inverse $(\neg p \rightarrow \neg q)$

I don't go to the beach whenever it is not a sunny summer day

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(c) When $\underbrace{\text{I stay up late}}_{p = \text{hypothesis}}$, $\underbrace{\text{it is necessary that I sleep until noon}}_{q = \text{conclusion}}$

Converse $(q \rightarrow p)$

It is necessary to stay up late whenever I sleep until noon

Contrapositive $(\neg q \rightarrow \neg p)$

If I don't sleep until noon, then I did not stay up late

Inverse $(\neg p \rightarrow \neg q)$

If I don't stay up late, then I do not sleep until noon

1.1.32

(a) $p \to \neg p$

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

(b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

contradiction!

(c) $p \oplus (p \vee q)$

p	q	(1) $p \vee q$	$p \oplus$ (1)
F	F	F	F
F	T	T	T
T	F	T	F
T	T	T	F

(d) $(p \wedge q) \to (p \vee q)$

p	q	(1) $p \wedge q$	(2) $p \vee q$	(1) $\to$ (2)
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

tautology!

(e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	(1) $q \to \neg p$	(2) $p \leftrightarrow q$	(1) $\leftrightarrow$ (2)
F	F	T	T	T	T
F	T	T	T	F	F
T	F	F	T	F	F
T	T	F	F	T	F

(f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q$

p	q	$\neg q$	(1) $p \leftrightarrow q$	(2) $p \leftrightarrow \neg q$	(1) $\oplus$ (2)
F	F	T	T	F	T
F	T	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T

tautology!!

1.2.32

Let's define the following variables

K_S ~ Smith is the killer	T_S ~ Smith tells the truth
K_W ~ Williams is the killer	T_W ~ Williams tells the truth
K_J ~ Jones is the killer	T_J ~ Jones tells the truth

Now we formulate some propositions from the information given.

$T_S \to \neg K_S$
$T_W \to \neg K_W$
$T_J \to \neg K_J$
} They all claim innocence

$K_S \to \neg T_S$
$K_W \to \neg T_W$
$K_J \to \neg T_J$
} Contrapositions

O_J ~ Jones was out of town

$T_J \to O_J$

$T_W \to \neg O_J$

C_J ~ Jones knew Cooper

$T_J \to \neg C_J$

$T_S \to C_J$

8 variables

(a) Some additional propositions

(i) one of the three men killed Cooper.

$$(K_J \wedge \neg K_W \wedge \neg K_S) \vee (\neg K_J \wedge K_W \wedge \neg K_S) \vee (\neg K_J \wedge \neg K_W \wedge K_S)$$

(ii) innocent men tell the truth:

$\neg K_S \to T_S \quad \neg K_W \to T_W \quad \neg K_J \to T_J$

(iii) guilty men may or may not lie.
can not draw info from this.

Case 1 $K_J \wedge \neg K_W \wedge \neg K_S$	Case 2: $\neg K_J \wedge \neg K_W \wedge K_S$	Case 3: $\neg K_J \wedge K_W \wedge \neg K_S$
This implies $T_W \wedge T_S$	This $T_J \wedge T_W$	$\to T_J \wedge T_S$
This $\neg O_J \wedge C_J$	which implies	$\to C_J \wedge \neg C_J$
More over $\neg T_J$	$O_J \wedge \neg O_J$	Contradiction
Jones is the killer	Contradiction.	

(b) Innocent cannot lie.

This case relaxes the condition that there must be one killer. We showed in (a) that only William and Smith can be innocent. So the only other possibility is that there is more than one killer. For instance there could be three killers. So we cannot tell who ~~be~~ are the killers.

1.3.8 (a) Kwame will not take a job in industry <u>and</u> will not go to graduate school.

(b) Yoshiko doesn't know JAVA nor CALCULUS.

(c) James old or weak.

(d) Rita will not move to Oregon nor Washington.

1.3.10 (b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	① $p \to q$	② $q \to r$	③ $p \to r$	④ ①∧②	④ → ③
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

tautology!

(1.3.10) Continued

(c) $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	(1) $p \rightarrow q$	(2) $p \wedge$ (1)	(2) $\rightarrow q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

tautology!

(d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

p	q	r	(1) $p \vee q$	(2) $p \rightarrow r$	(3) $q \rightarrow r$	(4) (1)$\wedge$(2)$\wedge$(3)	(4) $\rightarrow r$
F	F	F	F	T	T	F	T
F	F	T	F	T	T	F	T
F	T	F	T	T	F	F	T
F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

tautology!

(1.3.22) Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

$(p \to q) \wedge (p \to r)$ is true when p is false or all p, q, r are true. This is exactly the case when $p \to (q \wedge r)$ is true since p is false makes the implication $p \to (q \wedge r)$ true. Moreover if p is true this implication will require both q and r to be true.

(1.3.44) Show that $\neg$ and $\wedge$ are functionality complete.

If we assume (problem 42) that $\neg$, $\wedge$, and $\vee$ are functionally complete, it suffices to show that we can use $\neg$ and $\wedge$ to express any proposition $p \vee q$.

Consider an arbitrary proposition $p \vee q$.

By De Morgans $\neg(p \vee q) \equiv \neg p \wedge \neg q$

This $\neg\neg(p \vee q) \equiv \neg(\neg p \wedge \neg q)$

$(p \vee q) \equiv \neg(\neg p \wedge \neg q)$

Therefore $\vee$ can be expressed using $\neg$ and $\wedge$.

Problem 1.3.22 approach 2 (using laws)

$(p \to q) \wedge (p \to r)$

$\equiv (q \vee \neg p) \wedge (r \vee \neg p)$ star law

$\equiv \neg p \vee (q \wedge r)$ Demorgan law

$\equiv p \to (q \wedge r)$ star law

approach 3

(dirrect way : truth table)

p	q	r	$p \to q$	$p \to r$	$q \wedge r$	$p \to (q \wedge r)$	$(p \to q) \wedge (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

1.3.44

① "$\vee$" can be written in term of "$\neg$" and "$\wedge$":

$$p \vee q \equiv \neg\neg(p \vee q) \equiv \neg(\neg p \wedge \neg q)$$

② "$\rightarrow$" can be written in term of "$\neg$" and "$\wedge$":

$$p \rightarrow q \equiv q \vee \neg p \xrightarrow{\text{by ①}} \equiv \neg(\neg q \wedge \neg\neg p)$$
$$\equiv \neg(\neg q \wedge p)$$

③ "$\leftrightarrow$" can be written in term of "$\neg$" and "$\wedge$":

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \xrightarrow{\text{by ②}}$$
$$\equiv \neg(\neg q \wedge p) \wedge \neg(\neg p \wedge q)$$

④ "$\oplus$" can be written in term of "$\neg$" and "$\wedge$":

$$p \oplus q \equiv \neg(p \leftrightarrow q) \xrightarrow{\text{by ③}}$$
$$\equiv \neg\big(\neg(q \wedge p) \wedge \neg(\neg p \wedge q)\big)$$

Conclusion: Since all other logical operations can be written in term of "$\neg$" and "$\wedge$" so "$\neg$" and "$\wedge$" together are functionaly Complete.

1.3.50

p	q	$p \downarrow q$	$p \downarrow p$	$\neg p$	$(p\downarrow q)\downarrow(p\downarrow q)$	$p \vee q$
F	F	T	T	T	F	F
F	T	F	T	T	T	T
T	F	F	F	F	T	T
T	T	F	F	F	T	T

(a) (b)

To express any proposition with $\neg$, $\wedge$ and $\vee$

1. rewrite $\wedge$ using $\neg$ and $\vee$
2. rewrite $\vee$ using $(p\downarrow q)\downarrow(p\downarrow q)$
3. rewrite $\neg$ using $p \downarrow p$.

The resulting proposition will only use $\downarrow$.

1.50 other approach

Since $p \downarrow q$ is NOR by defenition (truth table)

it is $\neg(p \vee q)$

(a) $p \downarrow p \equiv \neg(p \vee p)$ defenition

$\equiv \neg p$

(b) $(p \downarrow q) \downarrow (p \downarrow q) \equiv \neg(p \downarrow q)$ using (a)

$\equiv \neg(\neg(p \vee q))$ defenition of $\downarrow$

$\equiv p \vee q$

(c) we need to rewrite all other operations

in term of $\downarrow$:

c-1 $\neg$: $\neg p \equiv p \downarrow p$ by part (a)

c-2 $\vee$: $p \vee q \equiv \neg(p \downarrow q)$ defenition

$\equiv (p \downarrow q) \downarrow (p \downarrow q)$

c-3 $\wedge$: $p \wedge q \equiv \neg(\neg(p \wedge q)) \equiv \neg(\neg p \vee \neg q)$

$\equiv \neg((p \downarrow p) \vee (q \downarrow q))$

$\equiv \neg(\underbrace{((p \downarrow p) \downarrow (q \downarrow q)) \downarrow ((p \downarrow p) \downarrow (q \downarrow q))}_{A})$

$\equiv A \downarrow A$

C-4 $\to$: $p \to q \equiv q \vee \neg p$

$$\equiv q \vee (p \downarrow p)$$

$$\equiv (q \downarrow (p \downarrow p)) \downarrow (q \downarrow (p \downarrow p))$$

C-5 $\leftrightarrow$: $$p \leftrightarrow q \equiv \underbrace{(p \to q)}_{\text{acording to C-4}} \underbrace{\wedge}_{\text{C-3}} \underbrace{(q \to p)}_{\text{C-4}}$$