1.4 1. Translation

6.f. Non of The students have visited N.D.

8.c. There is an animal such That if it's a rabbit, it hops.

10.a $\exists x \quad D(x) \wedge C(x) \wedge F(x)$

28.a-e

Let The Domain U be The set of all tools

Let The Predicates $CP(x)$, $EC(x)$ be "tool x is in The correct place" and "tool x is in excellent Condition" respectively

a: $\exists x \quad \neg CP(x)$

b: $\forall x \quad CP(x) \wedge EC(x)$

c: $\forall x \quad CP(x) \wedge EC(x)$

d: $\forall x \quad \neg(CP(x) \wedge EC(x))$

e: $\exists x \quad \neg CP(x) \wedge EC(x)$

2. Evaluation

12.b. $Q(-1) \equiv$ "$0 > -2$" True

12.e. "$\forall x \; x+1 > 2x$" False: Counterexample $x=1$

12.f. "$\exists x \; x+1 \leq 2x$" True: $x=1$ is an instance

14.d. $\forall x \; (2x > x)$ False: $x=0$ Counterexample

52.b $\exists! \, x \; (x^2=1)$ False: $x=1$ & $x=-1$ both satisfy the predicate while $\exists!$ means only one x

52.c $\exists! \, x \; (x+3=2x)$ True: The unique value of x that satisfies the predicate is 3

3. Free quantifiers from predicates

18.a $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$

18.d $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$

18.f $\neg \left(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)\right)$

20.e $\left(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)\right) \wedge \left((-5<0 \to P(-5)) \wedge (-3<0 \to P(-3)) \wedge (-1<0 \to P(-1)) \wedge (1<0 \to P(1)) \wedge (3<0 \to P(3)) \wedge (5<0 \to P(5))\right)$

20.e Continued

The rest of 20.e :

Though we can leave it, we can simplify also:

$$\begin{cases} -5 < 0 \to P(-5) \equiv P(-5) \\ -3 < 0 \to P(-3) \equiv P(-3) \\ -1 < 0 \to P(-1) \equiv P(-1) \end{cases} \qquad \begin{cases} 5 < 0 \to P(-5) \equiv T \\ 3 < 0 \to P(3) \equiv T \\ 1 < 0 \to P(1) \equiv T \end{cases}$$

Therefore the answer is equivalent to the following:

$$\big(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)\big) \wedge \big(\underbrace{P(-5) \wedge P(-3) \wedge P(-1)}_{A} \wedge T \wedge T \wedge T\big)$$

$$\equiv \big(A \wedge \neg P(-5)\big) \vee \big(A \wedge \neg P(-3)\big) \vee \big(A \wedge P(-1)\big) \vee \big(A \wedge P(1)\big) \vee \big(A \wedge P(3)\big) \vee \big(A \wedge P(5)\big) \quad \text{Identity \& Idempotent laws}$$

$$\equiv \big(P(-5) \wedge P(-3) \wedge P(-1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(5)\big)$$

$$\equiv \big(P(-5) \wedge P(-3) \wedge P(-1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3)\big) \vee \big(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(5)\big)$$

30.a $\quad P(1,3) \vee P(2,3) \vee P(3,3)$

30.b $\quad P(1,1) \wedge P(1,2) \wedge P(1,3)$

3.c $\quad \neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

3.d $\quad \neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

54. $\exists! x\, P(x) \equiv \exists x \underbrace{\left(P(x) \wedge \forall y\, (P(y) \rightarrow y = x) \right)}_{Q(x,y)}$

$$\exists! x\, P(x) \equiv \exists x\, Q(x,y)$$

$$\equiv Q(1,y) \vee Q(2,y) \vee Q(3,y)$$

$$\equiv \left(P(1) \wedge \forall y\, (P(y) \rightarrow y=1)\right) \vee$$

$$\left(P(2) \wedge \forall y\, (P(y) \rightarrow y=2)\right) \vee$$

$$\left(P(3) \wedge \forall y\, (P(y) \rightarrow y=3)\right)$$

$$\equiv \left(P(1) \wedge \left((P(1) \rightarrow \overset{T}{\cancel{1=1}}) \wedge (P(2) \rightarrow \overset{F}{\cancel{2=1}}) \wedge (P(3) \rightarrow \overset{F}{\cancel{3=1}})\right)\right)$$

$$\vee \left(P(2) \wedge \left((P(1) \rightarrow \overset{F}{\cancel{1=2}}) \wedge (P(2) \rightarrow \overset{T}{\cancel{2=2}}) \wedge (P(3) \rightarrow \overset{F}{\cancel{3=2}})\right)\right)$$

$$\vee \left(P(3) \wedge \left((P(1) \rightarrow \overset{F}{\cancel{1=3}}) \wedge (P(2) \rightarrow \overset{F}{\cancel{2=3}}) \wedge (P(3) \rightarrow \overset{T}{\cancel{3=3}})\right)\right)$$

$$\equiv \left(P(1) \wedge (T \wedge \neg P(2) \wedge \neg P(3))\right) \vee \left(P(2) \wedge (\neg P(1) \wedge T \wedge \neg P(3))\right)$$

$$\vee \left(P(3) \wedge (\neg P(1) \wedge \neg P(2) \wedge T)\right) \equiv$$

$$\left(P(1) \wedge \neg P(2) \wedge \neg P(3)\right) \vee \left(P(2) \wedge \neg P(1) \wedge \neg P(3)\right) \vee \left(P(3) \wedge \neg P(1) \wedge \neg P(2)\right)$$

notes on 54:

① $P \rightarrow T \equiv T$ & $P \rightarrow F \equiv \neg P$

② The answer is for review purpose. using The definition of $\exists!$ we could directly write The answer:

$\exists! x\, P(x)$ means only one x satisfies P Thus if 1 satisfies P, 2 & 3 must not satisfy P so $\neg P(2) \wedge \neg P(3)$ must be True. Same for 2 and 3:

$$\left(\underline{\underline{P(1)}} \wedge \neg P(2) \wedge \neg P(3)\right) \vee \left(\underline{\underline{P(2)}} \wedge \neg P(1) \wedge \neg P(3)\right)$$

$$\vee \left(\underline{\underline{P(3)}} \wedge \neg P(1) \wedge \neg P(2)\right)$$

4. Domain

22.c $U_1 = \{\text{Lionel messi}\}$

$U_2 = \{\text{lionel messi, cristiano ronaldo}\}$

with U_1 The Proposition is True.

with U_2 The Proposition is False.

another answer:

$U_1 = \{\text{Roberto Carlos, Roberto Baggio}\}$

U_2 = Students of Icom 4075

24.a - way 1 : Domain is students.

$\forall x \; C(x)$ which $C(x)$ is "x has Cellular"

way 2 : Domain is all People.

$\forall x \; S(x) \rightarrow C(x)$ which $S(x)$ is "x is student of my class"

24.b. way 1. Domain is students

$\exists x\ M(x)$ which $M(x)$ is "x has seen a movie"

way 2. Domain is all People.

$\exists x\ S(x) \rightarrow M(x)$

5 Negation of quantifiers

32 In all subsections of This question I assume The domain is The set of all animals.

32.a $\forall x$ (x is dog $\rightarrow$ x has fleas)

$\neg \forall x$ (x is dog $\rightarrow$ x has fleas)

$\equiv \exists x \neg$ (x is dog $\rightarrow$ x has fleas)

$\equiv \exists x$ (x is dog and (but) it doesn't have fleas)

note: $p \rightarrow q \equiv q \vee \neg p$ Thus $\neg(p \rightarrow q) \equiv \neg(q \vee \neg p)$

$\equiv \neg q \wedge \neg\neg p$

in Eng. There exists a dog That doesn't have fleas

$\equiv \neg q \wedge p$

32.b $\exists x$ (x is a horse and x can add)

$\neg \exists x$ (x is a horse and x can add)

$\equiv$ $\forall x \neg$(x is a horse and x can add)

$\equiv$ $\forall x$ (either x is not a horse or x cannot add)

in Eng. There is not any horse That can add.

note: recall you can first follow The rules to translate a predicate Then Paraphrase it to a fluent Eng.

32.c $\forall x$ (x is Koala $\rightarrow$ x can climb)

$\neg \forall x$ (x is Koala $\rightarrow$ x can climb)

$\equiv$ $\exists x \neg$(x is Koala $\rightarrow$ x can climb)

$\equiv$ $\exists x$ (x is Koala and x cannot climb)

Eng: There exit a koala That cannot climb.

32.d $\forall x$ (x is a monkey $\rightarrow$ x cannot speak F.)

or $\neg\exists x$ (x is a monkey and x can speak F.)

Negate of The second one : $\neg\neg\exists x$ (x is a monkey and x can speak F.)

$\equiv \exists x$ (x is a monkey and can speak F.)

Eng^1. There is a monkey That can speak French.

Negate of The first one : $\neg\forall x$ (x is monk. $\rightarrow$ x cannot speak F)

$\equiv \exists x \neg (\underbrace{x \text{ is monkey}}_{p} \rightarrow \underbrace{x \text{ cannot speak F.}}_{\neg q})$

$\equiv \exists x$ (x is a monkey and x can speak F.)

Eng^2. There is a monkey That can speak French

which of course is The same Thing

as Eng^1.

recall $\neg(p \rightarrow \neg q) \equiv \neg(\neg q \vee \neg p) \equiv q \wedge p$

32.e $\exists x$ (x is a pig and x can swim and x can fish)

$\neg \exists x$ (x is a pig and x can swim and x can fish)

$\equiv \forall x \neg$ (x is p and x can swim and x can fish)

$\equiv \forall x$ (x is not pig or x cannot swim or x can't fish

Eng. (different ways to say!) e.g

There is no any pig that can swim and fish

6.a Logical equivalence of predicates:

44. $\forall x\, (P(x) \leftrightarrow Q(x)) \overset{?}{\equiv} \forall x\, P(x) \leftrightarrow \forall x\, Q(x)$

The left proposition is always false because it claims that each two predicates are equivalent! of course there are predicates that are not. But the right proposition can be true sometimes (e.g. $P(x): 2x > 1$ & $Q(x): x > \frac{1}{2}$)

Therefore they are not equivalent.

Note on 44. Use priority of operations, quantifiers, and paranthesis to find something that acts last in each side. It would help to distinguish meannings of sides. For example here the left is a $\forall$ while the right is $\leftrightarrow$.

48. a yes. They are equivalent.

Since x does not appear as a free variable in A, A is a proposition that it's value does not depend on x. As any proposition A is whether T or F. If A is T, in both sides is T also for F.

Case 1 $A \equiv T$

$$\forall x\,(T \to P(x)) \equiv \forall x\, P(x) \equiv T \to \forall x\, P(x)$$

Case 2 $A \equiv F$

$$\forall x\,(F \to P(x)) \equiv \forall x\, T \equiv T \equiv F \to \forall x\, P(x)$$

48. b

Case 1 $A \equiv T$

$\exists x\,(A \to P(x)) \equiv \exists x\, P(x) \equiv A \to \exists x\, P(x)$

Case 2 $A \equiv F$

$\exists x\,(A \to P(x)) \equiv \exists x\, T \equiv T \overset{*}{\equiv} F \to \exists x\, P(x)$

$\equiv A \to \exists x\, P(x)$

note on equivalence $*$: $T \equiv F \to$ whatever

so whatever could be $\exists x\, P(x)$

50. $\forall x\, P(x) \vee \forall x\, Q(x) \not\equiv \forall x\,(P(x) \vee Q(x))$

using note on 44 the left is a $\vee$ while right is a $\forall$

Lets ~~prove be $\forall$ for each $x \in \mathbb{R}$~~ The domain be $\mathbb{R}$

and $P(x)$: x is positive $Q(x)$: x is negative or zero

right says each $x \in \mathbb{R}$ is eaither positive or negative or zero

left says "each $x \in \mathbb{R}$ is Positive" or "each $x \in \mathbb{R}$ is negative or zero"

50. Continued

for The defined circumestances right is T while left is F :

① « each $x \in \mathbb{Z}$ is positive » $\equiv F$

② « each $x \in \mathbb{Z}$ is negative or zero » $\equiv F$

$$① \lor ② \equiv F \lor F \equiv F$$

an equivalence in predicates most be held for any Domain and Predicate. This is not ~~equivalent~~ equivalence because at least for The Domain $\mathbb{Z}$ and P & Q above They are not always ~~equivalent~~ with the same truth value (never ~~equivalent~~ have the same truth value in This ~~case~~ example indeed); so they are not equivalent.

(1.5) 7. Translation of nested quantifiers.

4.a. There exist a student in The class who has taken a Computer science Course.

« note: a could be some or at least one »

4.b. There is a student in The class who has taken all Computer science Courses.

4.c. all students of The class have taken at least one course.

« note: That doesn't mean They have taken some course »

4.d There is a Computer science Course that has been taken by all students in The class.

« note: $\exists y \forall x P(x,y) \equiv \exists x \forall y P(y,x)$ »

The note says that The names of variables are not important But The order of variables in $P(x,y)$ according to definition of P.

4.e each computer science course has been taken by at least one student in the class.

4.f. ① all students in the class have taken all computer science courses. or

② all computer science courses has been taken by all students in the class.

①: $\forall x \forall y\ P(x,y)$ ② $\forall y \forall x\ P(x,y)$

① $\equiv$ ②

6.a Randy Goldberg has enrolled in class CS 252

6.b There exit a student in the school that is enrolled in Math 695

6.c There exist a class y that carol is enrolled in it

6.d There is a student who is (remember we can say: some students are...) enrolled in Math222 and CS252

6.e $\exists x \exists y \forall z \left((x \neq y) \wedge \left(C(x,z) \rightarrow C(y,z) \right) \right)$

answer 1. There are some students for them there are some other students who whatever course that they are enrolled in, those other students also are enrolled in.

answer 2. There is a pair of different students x and y such that if x is enrolled in course z y is ~~also~~ enrolled in z either.

notes: ① x and y are students because in $C(x,z)$ and $C(y,z)$ they came first. z is a course since is second.

② There are several other way to translate this proposition.

6.f. There are some students in the school who for them there is at least one other student that is enrolled in the same class.

10.a $\forall x \; F(x, \text{Fred})$

10.b $\forall x \; F(\text{Evelyn}, x)$

10.c $\forall x \exists y \; F(x,y)$

10.d $\forall x \exists y \; \neg F(x,y)$

10.e $\forall x \exists y. \; F(x,y)$

10.f $\forall x \; \neg F(x, \text{Fred}) \vee \neg F(x, \text{Jerry})$

10.g $\exists x_1, x_2 \; F(\text{Nancy}, x_1) \wedge F(\text{Nancy}, x_2) \wedge$
$\forall y \notin \{x_1, x_2\} \; \neg F(\text{Nancy}, y)$

22. $\exists x \in \mathbb{Z}^+ \; \underbrace{\forall x_1, x_2, x_3}_{\equiv \forall x_1 \forall x_2 \forall x_3} \in \mathbb{Z}^+ \; x \neq x_1^2 + x_2^2 + x_3^2$

30.c $\neg \exists y \, (Q(y) \wedge \forall x \neg R(x,y))$

$\equiv \forall y \, \neg (Q(y) \wedge \forall x \neg R(x,y))$

$\equiv \forall y \; \neg Q(y) \vee \neg \forall x \neg R(x,y)$

$\equiv \forall y \; \neg Q(y) \vee \exists x \neg\neg R(x,y)$

$\equiv \forall y \; \neg Q(y) \vee \exists x \; R(x,y)$

30.e $\neg \exists y \, (\forall x \exists z \, T(x,y,z) \vee \exists x \forall z \, U(x,y,z))$

$\equiv \forall y \neg (\forall x \exists z \, T(x,y,z) \vee \exists x \forall z \, U(x,y,z))$

$\equiv \forall y \, (\neg \forall x \exists z \, T(x,y,z) \wedge \neg \exists x \forall z \, U(x,y,z))$

$\equiv \forall y \, (\exists x \forall z \neg T(x,y,z) \wedge \forall x \exists z \neg U(x,y,z))$

32.c $\neg \exists x \exists y \, (Q(x,y) \leftrightarrow Q(y,x))$

$\equiv \forall x \forall y \, \neg (Q(x,y) \leftrightarrow Q(y,x))$

$\equiv \forall x \forall y \, (Q(x,y) \oplus Q(y,x))$

32.d $\neg \forall y \exists x \exists z \big(T(x,y,z) \vee Q(x,y) \big)$

$\equiv \exists y \, \forall x \, \forall z \, \neg \big(T(x,y,z) \vee Q(x,y) \big)$

$\equiv \exists y \, \forall x \, \forall z \;\; \neg T(x,y,z) \wedge \neg Q(x,y)$

36.b ~~$\exists x$~~ The domain is set of students in The class.

~~$\exists x$~~ $C(x,y)$: student x chatted with student y

$\exists x \exists y \; \big(C(x,y) \wedge \forall z \neq y \; \neg C(x,z) \big)$

9. Domain for nested quantifiers

34. $U_1 = \{2,3\}$

$U_2 = \mathbb{R}$ all real numbers

46.c True (for $x \leq 0$)

46.b True (for $x = 0$)

46.a False (because $\forall x \in \mathbb{R}^+ - \{0\}$ $\exists y \in (0, \min\{1, \frac{1}{x}\})$ $y^2 < x$)