09-25-14

① $A \cup A = A$ (Prove it)

② $A = \{a, \{a\}, \{a, \{a\}\}\} \to |A| = 3$

③ $B = P(P(\phi))$ $|B| = ?$

$|B| = |P(P(\phi))| = 2^1 = 2$

$|Pow(\phi)| = 2^0 = 1$

$|\phi| = 0$

⑤ $A \times B = \emptyset \to A = \emptyset \vee B = \emptyset$ (Prove it)

[redacted]

→ Proof by Contrapositive

$A \neq \emptyset \wedge B \neq \emptyset \to A \times B \neq \emptyset$

[redacted] way1 to write:

① $A \neq \emptyset$ premise

② $\neg A = \emptyset$ equivalent

③ $\exists x_1, x_1 \in A$ def. of empty

④ $B \neq \emptyset$ premise

⑤ $\exists x_2$ $x_2 \in B$

⑥ $(x_1, x_2) \in A \times B$

⑦ $\exists (a,b)$ $(a,b) \in A \times B$ E.G.

⑧ $A \times B \neq \emptyset$

other way: $A \neq \emptyset \to \exists x$ $x \in A$

$A \neq \emptyset \to \exists y$ $y \in B$

Lets a be the element in A and b be the element in B. So (a,b) must be in $A \times B$ according to the definition of $A \times B$ then $A \times B \neq \emptyset$

$P(A) \subset P(B) \leftrightarrow A \subset B$

① $\underline{A \subset B}$ premise.

② $X \in P(A) \to X \subset A \to$

$\to X \subset B \to X \in P(B)$

$P(A) \subset P(B)$

$P(A) \subset P(B)$ premise

$x \in A \to \{x\} \in P(A)$

$\to \{x\} \in P(B) \to \{x\} \subset B$

$\to x \in B$

$A \subset B$

$x \notin B \quad x \notin A$

$A \subset B \leftrightarrow \bar{B} \subset \bar{A}$

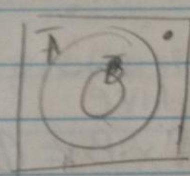
$A \subset B$

$\bar{B} \subset \bar{A}$

$\forall x \in A \to x \notin \bar{A}$

$\to x \notin \bar{B} \to \{B \subset A\}$

$x \in B$



contrapositive

$\bar{B} \subset \bar{A}$

$\forall_x \quad x \in \bar{B} \to x \in \bar{A}$

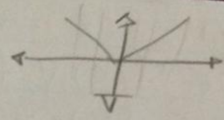
$\Leftrightarrow \forall_x \quad x \notin \bar{A} \to x \notin \bar{B}$

other part.

$A \subset B$

$x \in \bar{B} \to x \notin B \xrightarrow{A \subset B} x \notin A \to x \in \bar{A}$

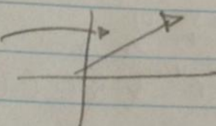
xample.



$f(x) = |x|$

$f: \mathbb{R} \to \mathbb{R}$

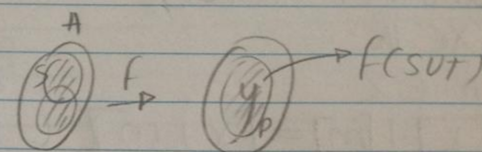
$f: \mathbb{R}^{+} \to \mathbb{R}$



② $f: A \to B$

$S, T \subset A$

$f(S \cup T) = f(S) \cup f(t)$



$f(x) = y$

Solution

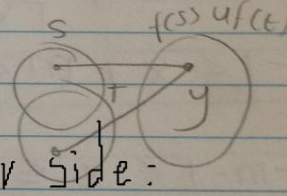
$y \in f(S \cup T) \to \exists_x \in S \cup T \,;\, f(x) = y$

$\to x \in S \lor x \in T$

$\to f(x) \in f(S) \lor f(x) \in f(T)$

$\to f(x) \in f(S) \cup f(T)$

$\to y \in f(S) \cup f(T)$ other side:



$y \in f(S) \lor y \in f(T)$

$\to \exists x \in S \; f(x) = y \lor \exists x' \in T \; f(x') = y$

$$\left.\begin{array}{l} x \in S \to x \in S \cup T \\ x' \in T \to x' \in S \cup T \end{array}\right\} \to y = \underset{\uparrow \atop S \cup T}{f(x)} = f(x') \in f(S \cup T)$$

$x' \leftarrow x$ prime (another element)

$y \in f(S \cup T) \rightarrow \exists x \in S \cup T;\ f(x) = y$

$x \in S \vee x \in T \rightarrow f(x) \in f(S) \vee f(x) \in f(T)$

$\rightarrow f(x) \in f(S) \cup f(T) \rightarrow y \in f(S) \cup f(T)$

$y \in f(S) \cup f(T) \rightarrow y \in f(S) \vee y \in f(T)$

if $y \in f(S) \rightarrow \exists x \in S \quad f(x) = y \xrightarrow{x \in S \rightarrow x \in S \cup T} \exists x,\ x \in (S \cup T) \rightarrow y = f(x) \in f(S \cup T)$

if $y \in f(T) \rightarrow \exists x' \in T \quad f(x') = y \xrightarrow{x' \in S \rightarrow x' \in S \cup T} \exists x',\ x' \in (S \cup T) \rightarrow y = f(x') \in f(S \cup T)$

$y \in f(S \cup T)$

another exercise:

$x \in \mathbb{R}$

$m \in \mathbb{Z}$

$\lceil x \rceil + \lceil m \rceil = \lceil x + m \rceil$

definition of $\lceil\ \rceil$

properties of $\lceil\ \rceil \lfloor\ \rfloor$

ceil floor

Solution:

$\lceil x + m \rceil = y \rightarrow y \in \mathbb{Z}$

$y - 1 < x + m \leq y$

$y - m - 1 < x \leq y - m$

$\lceil x \rceil = y - m$

$\lceil x + m \rceil = y \xrightarrow{\text{def}} y - 1 < x + m \leq y$

$\rightarrow y - m - 1 < x \leq y - m$

$\rightarrow \lceil x \rceil = y - m \rightarrow \lceil x \rceil = \lceil x + m \rceil - m$

$\rightarrow \lceil x \rceil + m$

$= \lceil x + m \rceil$