15. a. $r \wedge \neg p$

b. $\neg p \wedge q \wedge r$

c. $r \to (q \leftrightarrow \neg p)$

in the
case q then

$(r \to q) \leftrightarrow \neg p$

$\to (\neg r \vee q)$

① apply parenthesis.

② Grizzily Bears have not been seen in the area if and only if hiking is safe on the trail and Berries are not ripe along the trail.

27

b. quiz $\to$ class

converse: If class, then quiz.

inverse: If no quiz, then no cla

contra: If no class, then no quiz.

p only if q.

c. Conv: If no diviser then posi integer.

(33) a) $(p \vee q) \to (p \oplus q)$

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

* learn tabla de exclusive or

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

p	q	r	$\neg p$	$\neg r$	★ $p \leftrightarrow q$	♡ $\neg p \leftrightarrow \neg r$	★ $\oplus$ ♡
T	T	T	F	F	T	T	F
T	T	F	F	T	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	[illegible]

9.a

$p \wedge q \rightarrow p$

p	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

10) a. $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

21) $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$

equivalent!!

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	p	q	$\neg p$	$\neg p \leftrightarrow q$
T	T	T	F	T	T	F	F
T	F	F	T	T	F	F	T
F	T	F	T	F	T	T	T
F	F	[illegible]	F	F	F	T	F

24) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

left side

$(q \to r) \vee p$

$(r \vee \neg q) \vee p$

$(p \vee r) \vee \neg q \longleftarrow$ associative

$\boxed{q \to (p \vee r)}$

$\neg q \vee (p \vee r)$

$p \to q \equiv \neg p \vee q$

$$\neg(p \leftrightarrow q) \overset{?}{\equiv} \neg p \leftrightarrow q$$

$$\neg(p \leftrightarrow q) \equiv \neg((p \to q) \land (q \to p))$$

$$\neg(p \to q) \lor \neg(q \to p)$$

$$\neg(q \lor \neg p) \lor \neg(p \lor \neg q)$$

$$(\neg q \land p) \lor (\neg p \land q)$$

$$[(\neg q \land p) \lor \neg p] \land [(\neg q \land p) \lor q]$$

$$[(\neg p \lor \neg q) \land \underbrace{(\neg p \lor p)}_{T}] \land [\underbrace{(q \lor \neg q)}_{T} \land (q \lor p)]$$

Entonces usando $(\neg p \leftrightarrow q)$

$(\neg p \to q) \land (q \to \neg p)$

$(q \lor p) \land (\neg p \lor \neg q)$

$$\equiv (\neg p \lor \neg q) \land (q \lor p)$$

$$\equiv (q \to \neg p) \land (\neg p \to q)$$

$$\equiv q \leftrightarrow \neg p$$

OK

(41) 13 → $S = \{\neg, \wedge\}$ → or ↔ y ∧ 09-09-19

now any operation we can write it in terms of $\{\neg, \wedge\}$ fix

Section 1.4

6e. $\neg \forall x\, N(x)$

$\exists x\, \neg N(x)$

↳ There exists a student that hasn't visited North Dakota.

6c. $\neg \exists x\, N(x)$

$\forall x\, \neg N(x)$

No ↳ None of the students has visited North Dakota.

↳ For any student in the class, the student hasn't visited N.D.

19a. $\exists x(P(x) \wedge Q(x))$

9b. $\exists x(P(x) \wedge \neg Q(x))$

9d. $\forall x\, \neg(P(x) \wedge Q(x))$ (Book says $\forall x \neg(P(x) \vee Q(x))$)

29a. Domain → propositions

$T(x) \rightarrow x$ is a Tautology

$\exists x(T(x))$

ba → a) $Q(0) \rightarrow T$

f) $\exists x \neg(Q(x)) \rightarrow T$

14a. $\exists x(x^3 = -1) \rightarrow T$

14c. $\forall x((-x)^2 = x^2) \rightarrow T$

18c. $D = -2, -1, 0, 1, 2$

c) $\exists x \neg P(x)$

$\to \neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

20c. $\forall x ((x \neq 1) \to P(x))$ Domain
-5
-3
-1
1

$(-5 \neq 1) \to P(-5)) \wedge$
$(-3 \neq 1) \to P(-3) \wedge$
$(-1 \neq 1) \to P(-1) \wedge$
$\underbrace{(1 \neq 1) \to P(1)}_{T}$
F

52a F
52d F

22b. $\exists x$ $age(x) > 21$
$U_1 = \{$ any person $\to T \}$
$U_2 = \{$ the students in Kindergarden $\}$

~~T(x) = x can learn new tricks.~~

~~D → old dogs.~~

33a

U = dogs

$\exists x$ (x is old → x can learn)

$\forall x \neg(\underbrace{\text{x is old}}_{p} \rightarrow \underbrace{\text{x can learn}}_{q})$

$\forall x \neg(\neg p \vee q)$

$\forall x (p \wedge \neg q)$

$\forall x$ (x cannot learn $\wedge$ x is old)

* All the dogs are old and cannot learn new tricks.

NO!

√ For any x inside parenthesis it is always true

(43) $\forall x(P(x) \rightarrow Q(x)) \not\equiv \underbrace{\forall x P(x)}_{T,F} \rightarrow \underbrace{\forall x Q(x)}_{F}$

counter example:

$Q(x) \equiv F$ $P(x) =$

sometimes once; sometimes false

always F

$\boxed{\text{Section 1.5}}$

30d. $\neg\exists y(\exists x R(x,y) \vee \forall x S(x,y))$

$\forall y(\neg(\exists x R(x,y)) \wedge \neg\forall x S(x,y))$

$\forall y(\forall x \neg R(x,y)) \wedge \exists x \neg S(x,y))$

35: a) $\forall x (x^2 \geq x)$

Sec 1.4 $x = \frac{1}{2}$ counter example

b) $\forall x (x > 0 \lor x < 0)$

$x = 3$

c) $\forall x (x = 1)$

$x = 2$

Sec 1.5

35 $\underbrace{\forall x \forall y \forall z}\ \exists w \neq_{x,y,z}$

$U = \mathbb{R}$ 1 12 90 true cause I can choose 3

$U = \{1, 2, 3\}$ I cannot choose a w diff From the given x, y, z False

---

7d. $\forall x \forall z \exists y ((x \neq z) \rightarrow \neg(T(x,y) \land T(z,y)))$

$T(x, y)$ $\uparrow$ student $\uparrow$ cuisine $T(z,y)$ $\uparrow$ H is a student

For any two students, there exists a cuisine, that if x and z are not the same then the first student doest not like the cuisine or the second studen does not like

Fix → For any two different students there is a food that at least one of them does not like.

Reunion
I COM.
September 16

$\forall x\,(\overbrace{A \to P(x)}^{T})$
depends now
on $\forall x\,(P(x) \to$

Section 1.6

9a. P= K live in Aus.
q = K are M.

$$\frac{p \wedge q}{\therefore q}\ \Big\}\ \text{simplification}$$

b. P= Hotter than 100.
q= Pollution is dangerous.

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}\ \Big\}\ \text{disjunctive syllogism}$$

10a. I play hockey = p
I am sore the next day = q
I use whirpool = r

$$\begin{array}{l} p \to q \\ q \to r \\ \neg r \\ \hline \therefore \neg p \end{array} \qquad p \to r,\ \neg r$$

~~por~~
solución del profe:

1. $q \to r$ premise
2. $\neg r$ " "
3. $\neg q$ from 1 & 2
4. $p \to q$ premise
5. $\therefore \neg p$ from 3 & 4

11. A_2

$$\frac{p_1, p_2 \ldots p_n}{\therefore q \to r}$$

A_1

$$\frac{p_1, p_2, \ldots, p_n, q}{\therefore r}$$

if A_1 is valid, then A_2 is valid.

$(p_1, p_2, .. p_n) \to (q \to r)$

Let's A_2 is not valid then it is possible that $q \to r$ is F. There is only one case in which $q \to r$ is F which is $q \equiv T$ and $r \equiv F$

in this case using A_1 having $p_1, p_2, \ldots p_n, q$ we have $r \equiv T$. r cannot be T and F at same time.

19b. $n = \mathbb{R}$

$p \to q$ $\quad n > 3 \to n^2 > 9$

$\neg q$ $\quad n^2 \le 9$

$\therefore \neg p$ MT $\quad n \le 3$

valid!

19.c $n = \mathbb{R}$ $\quad n > 2 \to n^2 > 4$

$n \le 2$

$\therefore n^2 \le 4$

not valid!

$p \to q$ $\quad q \vee \neg p$

$\neg p$ $\quad \neg p$

$\neg q \vee q \equiv T$ $\quad T$

25 Solution

$\forall x (P(x) \to Q(x))$

$\neg Q(a)$

$\therefore \neg P(a)$

1. $\forall x (P(x) \to Q(x))$ Premise
2. $p(a) \to q(a)$ UI
3. $\neg Q(a)$ premise.
4. $\neg P(a)$ 2, 3, MT

27 Show.

$\forall x (P(x) \to Q(x) \wedge S(x)))$

$\forall x (P(x) \wedge R(x))$

$\therefore \forall x (R(x) \wedge S(x)))$

1. $\forall x (P(x) \to (Q(x) \wedge S(x)))$ Premise

$P(a) \to Q(a) \wedge S(a)$. UI

$\forall x (P(x) \wedge R(x))$ Premise

$P(a) \wedge R(a)$ UI

$P(a)$

$R(a)$

$Q(a) \wedge S(a)$ $\quad q(a) \to s(a)$

$R(a) \wedge S(a)$

$\forall x (R(x) \wedge S(x))$ UG

section 1.7.

④ $n = \mathbb{R}$

* If n is even then -n is even.

inference rule:

$\Rightarrow n = 2K \Rightarrow -n = -2K \Rightarrow \underbrace{-n = 2(-K)}$ $\forall_{a,b}\,(a = b \Rightarrow -a = -b)$

$$\frac{P(n, 2K)}{\therefore\ Q(n, 2K)}$$

* product of two rational #s is rational

$a = \frac{p_1}{q_1}$ $p_1 q_1 \in \mathbb{R}$, $q_1 \neq 0$

$b = \frac{p_2}{q_2}$ $p_2, q_2, \in \mathbb{R}$, $q_2 \neq 0$

$a \cdot b$ is rational. $\exists$

$\exists\, P_3, q_3 \neq 0$ $a \cdot b = \frac{P_3}{q_3}$

$a \cdot b = \frac{P_1 P_2}{q_1 q_2}$

. since q_1 and $q_2 \neq 0$ then

• the product of two integer #s is integer

so $p_1 p_2 \in \mathbb{Z}$, $q_1 q_2 \in \mathbb{Z}$

$p_3 = p_1 p_2$ $q_3 = q_1 q_2$

$a \cdot b = \frac{p_3}{q_3}$

(19)

(20) prove $p(0)$, where $p(n)$ is the proposition "If n is a positive integer >1, then $n^2 \geq n$."

Prove $P(0) \equiv T$

$\boxed{0 > 1 \rightarrow \quad 0^2 > 0}$ ← Proof

vacuous.

rule
$p \rightarrow q$
$\neg p$
$\therefore \neg q$

(20) $n \in \mathbb{R}^+$

n is even $\longleftrightarrow$ $7n+4$ even

$n = (2K) \Rightarrow 7(2K) + 4$

$= 2(7K) + 2 \cdot 2$

$= 2(7K + 2)$

$= 2t$

$\boxed{t \in \mathbb{R}}$

contradiction: $7n + 4 = 2t \Rightarrow 3K \qquad n = 2K$

let's $n = 2K+1$

$7n + 4$

$7(2K+1) + 4$

$14K + 11$

$14K + 10 + 1$

$2\underbrace{(7K+5)}_{t} + 1$

$2t + 1$ ← uneven

30 P_1, P_2, P_3, P_4

$\left.\begin{matrix} P_1 \\ P_2 \end{matrix}\right\} P_1 \equiv P_2$ $\left.\begin{matrix} P_1 \equiv P_2 \\ P_3 \equiv P_4 \end{matrix}\right\} P_2 \equiv P_3$

$\left.\begin{matrix} P_3 \\ P_4 \end{matrix}\right\} P_3 \equiv P_4$

1.8

4 $\min(a, \min(b,c)) = \min(\min(a,b), c)$

case 1: $a \le b \le c \rightarrow$ left side $= \min(a,b) = a$, right side $= \min(a,c) = a$

case 2: $a \le c \le b$

" 3: $b \le a \le c$

" 4: $c \le a \le b$

" 5: $b \le c \le a$

" 6: $c \le b \le a$

$\min(x,y) = (x + y - |x-y|)/2$

$x<y$ $\quad x + y - (y-x)$

$y<x$ $\quad (x+x)/x$

x

$|x-y| = y-x$

because if

$x<y$

$x-y<0$

$|x-y| = -(x-y)$

$\boxed{y-x}$