$c = 11b \bmod 13$

$c \bmod 13 = 11b \bmod 13$

$c \bmod 13 = 11(9 \bmod 13) \bmod 13$

$c \bmod 13 = 99 \bmod 13 \bmod 13$

$c \bmod 13 = 8 \bmod 13$

$$\begin{array}{r} 7 \\ 13\overline{)99} \\ \underline{91} \\ 8 \end{array}$$

4.1 16

$a \bmod m = b \bmod m) \leftrightarrow a \equiv b \bmod m$

$m \mid a-b$

$\exists c \;\; a-b = cm$

$a \bmod m = r$

$\exists q, r \;\; a = qm + r$

$a \bmod m = r_1$

$\exists q_1 \;\; a = q_1 m + r_1 \rightarrow a - q_1 m = r_1$

$b \bmod m = r_2$

$\exists q_2 \;\; b = q_2 m + r_2 \rightarrow b - q_2 m = r_2$

$r_1 = r_2 \rightarrow a - q_1 m = b - q_2 m$

$a - b = (q_1 - q_2)m \longrightarrow a = b \bmod m$

$c = q_1 - q_2$

$f \in \Omega(g) \to \exists c \quad cg < f$

$g \in \Theta(h) \to g \in \Omega(h)$

$g \in \Omega(h) \to \exists c' \quad c'h < g$

$cg < f \to cc'h < f$

$\exists cc' \quad cc'h < f$

$\to f \in \Omega(h)$

4.1 13) find c $\quad 0 \leq c \leq 18$

$a = 4 \pmod{13}$

$b = 9 \pmod{13}$

a) $c \equiv 9a \pmod{13}$

$c \bmod 13 = 9 \bmod 13$

$c(4) = 9 \cdot 4$

$\frac{c(4)}{4} \quad \frac{36}{4}$

$c = 9$

$13 \mid 9 - 4$

$c \bmod 13 = 9a \bmod 13$

$c \bmod 13 = 9(4 \pmod{13}) \bmod 13$

$c \bmod 13 = 36 \bmod 13 \bmod 13$

$c \bmod 13 = 10 \pmod{13}$

$$\begin{array}{r} 2 \\ 13\overline{)36} \\ \underline{26} \\ 10 \end{array}$$

Recitation Class 10/21/14

$n^2 \notin O(n\log^2 n)$

$$A = \lim \frac{n^2}{n\log^2 n} = \frac{\infty}{\infty}$$

$$\frac{n}{\log^2 n} = \frac{1}{\frac{\log^2 n}{n}} = \frac{n}{2\log n} = \frac{1}{2\frac{1}{n}} = \frac{n}{2} = \infty$$

$n^2 \in \Omega(n\log^2 n)$

$n\log^2 n \in O(n^2)$

(14)
$$\frac{\begin{array}{c} f(x) \in \Theta(g(x)) \\ g(x) \in \Theta(h(x)) \end{array}}{f(x) \in \Theta(h(x))} \longrightarrow f \in O(h) \wedge f \in \Omega(h)$$

$f \in \Theta(g) \rightarrow f \in O(g)$

$f \in O(g) \rightarrow \exists c \;\; f < cg$

$g \in \Theta(h) \rightarrow g \in O(h)$

$g \in O(h) \rightarrow \exists c \;\; g < c'h$

$f < cg \Rightarrow f < cc'h$

$\exists cc' \;\; f < cc'h \rightarrow f \in O(h)$

$p \to q \qquad \neg q \to \neg p$

$a = b \bmod m$

$m \mid a-b$

$\exists c \quad a-b = cm$

let's $r_1 \neq r_2$

$r_1 = a - q_1 m$

$b \bmod m = r_2$

$r_2 = b - q_2 m$

$\Rightarrow a - q_1 m \neq b - q_2 m$

$a - b \neq (q_1 - q_2) m$

4.3 50

$a, b, m \in \mathbb{Z} \qquad m \geq 2$

$a = b \bmod m \to \gcd(a,m) = \gcd(b,m)$

$\gcd(a,m) = a_1 \to \exists c \quad a = c_1 a_1 \qquad a_1 = \frac{a}{c_1}$

$\gcd(b,m) = b_1 \to \exists c_2 \quad m = c_2 a_1 \qquad a_2 = \frac{m}{c_2}$

$\frac{b}{c_1} = \frac{m}{c_2} \qquad \frac{a}{c_1} = \frac{m}{c_2}$

$a \equiv b \bmod m \to m \mid a-b \to \exists c \quad mc = a-b$

$mc = \frac{c_1}{c_2} m - \frac{c'}{c'_2} m \qquad c = \frac{c'}{c_2} - \frac{c'_1}{c'_2}$