Recitation Class

4). Let P(n) be the stame ...

a) $P(1) = (1(1+1/2))^2 = 1$

b) $P(1) = \left(\frac{1(1+1)}{2}\right)^2 = 1^3 = 1$

$P(k) = 1^3 + 2^3 + \dots + k^3 \quad \left(\frac{k(k+1)}{2}\right)^2$

$P(k+1) = \left(\frac{(k+1)(k+2)}{2}\right)^2$

$P(k+1) = 1^3 + 2^3 + (k+1)^3$

$P(k) + (k+1)^3 =$

$\left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$

$\frac{k^2}{2^2} (k+1)^2 + (k+1)^2 (k+1)$

$(k+1)^2 \left(\frac{k^2}{4} + (k+1)\right.$

$(k+1)^2 \left(\frac{k^2+4k+4}{4}\right)$

$(k+1)^2 \frac{(k+2)^2}{4} = \left(\frac{(k+1)(k+2)}{2}\right)^2$

(8) $\underbrace{2 - 2\cdot 7}_{n=1} + 2\cdot 7^2 - \dots + 2(-7)^n = \frac{1-(-7)^{n+1}}{4}$

$P(1) = 2-(1+1)^1 = \frac{1-(-7)^2}{4}$

$-12 = -12$ $\qquad 2-2(7)+\dots+2-7^5+$

$P(1) = T$

$P(k) = \frac{1-(-7)^{k+1}}{4}$ $\qquad P(k+1) \overset{?}{=} \frac{1-(-7)^{k+1+1}}{4}$

$P(k+1) = \frac{1-(-7)^{k+1}}{4} + 2(-7)^{k+1}$

$\frac{1-(-7)^{k+1}+8\cdot(-7)^{k+1}}{4}$

$\frac{1+7(7)^{k+1}}{4} = \frac{1=(-7)^{k+2}}{4}$

(9)

20 $P(n): 3^n < n! \quad \forall n \geq 6$

$P(7) = 2187 < 5040$

$P(k+1) = 3^{k+1} < (k+1)!$

$3^k \cdot 3 < k! \cdot 3 < k!\,(k+1) = (k+1)!$

$3^{k+1} < (k+1)!$

31 $2 \mid n^2 + n \quad \forall n > 0$

$P(1) \quad 2 \mid 1+1 = T$

$P(k+1) \quad 2 \mid (k+1)^2 + k+1$

$k^2+2k+1+k+1 \Rightarrow k^2+k + 2k+1$

$P(k) = 2 \mid k^2+k$

$2 \mid 2(k+1)$

$2 \mid k^2+k+2k+1 \Rightarrow 2 \mid (k+1)^2+(k+1)$

(38) $A_1, A_2 \ldots A_n$ / $B_1, B_2 \ldots B_n$ $\}$ $\forall j > 0$ $A_j \subseteq B_j$

$$P(n) = \bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j.$$

lemma if $A_1 \subseteq B_1$ and $A_2 \subseteq B_2$ Then $A_1 \cup A_2 \subseteq B_1 \cup B_2$

$x \in A_1 \cup A_2 \rightarrow x \in A_1 \vee x \in A_2$

if $x \in A_1$, then $x \in B_1$ because $A_1 \in B_1$, so $x \in B_1 \cup B_2$

$x \in A_2$ then $x \in B_2$ because $A_2 \in B_2$ so $x \in B_1 \cup B_2$

$$P(1) = A_1 \subseteq B_1 = T$$

$$P(k+1) \quad \bigcup_{j=1}^{k+1} A_j \subseteq \bigcup_{j=1}^{k+1} B_j$$

$$\bigcup_{j=1}^{k+1} A_j = \bigcup_{i=1}^{k} A_j \cup A_{k+1} \subseteq \bigcup_{i=1}^{k} B_j \cup B_{k+1}$$

because of lemma.

$$= \bigcup_{j=1}^{k+1} B_j$$

(12) $\forall(n)\ P(n): n\ \sum_{i\in S} 2^i \quad S \subseteq 2^+$

$P(1) = 1 = 2^0$

$P(k+1) = k+1\ \sum_{i\in S \subseteq 2^+} 2^i$

Case 1: $k+1$ is even then $\frac{k+1}{2}$ is integer

$\frac{k+1}{2} = \sum_{i\in S_1} 2^i = 2\,\frac{k+1}{2} = 2\sum_{i\in S} 2^i$

$k+1 = \sum_{i\in S_1} 2^{i+1} = \sum_{i\in 2S} 2^j$

$P(k+1) = T$

Case 2 $k+1 =$ odd. then $k =$ ~~integer~~ even

$\frac{k}{2} = \sum_{i\in S} 2^i$

$2\cdot\frac{k}{2} = 2\sum_{i\in S} 2^i = \sum_{i\in S_1} 2^{i+1}$

$k+1 = \sum 2^i$