5.1/#6 $1 \cdot 1! + 2 \cdot 2! \ldots n \cdot n! = (n+1)! - 1$

$P(1)$ true since $1 \cdot 1! = (1+1)! - 1$

$1 = 1$

$P(k) \to P(k+1)$ $1 \cdot 1! + 2 \cdot 2! \ldots k + (k+1)! + (k+1)(k+1)!$

$(k+1)! - 1 + (k+1)(k+1)!$

$(k+1)! [1 + k + 1] - 1$

$(k+1)! (k+2) - 1$

$(k+2)(k+1)! - 1$

$(k+2)! - 1$

5.1/8

$P(0)$ $2(-7)^0 = \frac{1-(-7)^1}{4}$

$2 = \frac{8}{4}$

$2 = 2$

$P(k) \to P(k+1)$

$$2 - 2 \cdot 7 + 2 \cdot 7^2 - \ldots + 2(-7)^k + 2(-7)^{k+1} = \frac{1-(-7)^{k+1}}{4} + 2(-7)^{k+1}$$

$$2(-7)^k + 2(-7)^{k+1} = \frac{1-(-7)^{k+1} + 8(-7)^{k+1}}{4}$$

$$2(-7)^k + 2(-7)^{k+1} = \frac{1 + (-7)^{k+1}(8-1)}{4}$$

$$= \frac{1 + (-7)^{k+1}(7)}{4}$$

$$= \frac{1 - (-7)^{k+1}(-7)}{4}$$

$$= \frac{1-(-7)^{k+2}}{4}$$

5.1/20

$P(n): 3^n < n!$

$\forall n > 6$

$P(7)\ 3^7 < 7!$ true

$P(k) \to P(k+1)$

$3 \cdot 3^k < k! \cdot (k+1)$

$3^{k+1} < k!(k+1)$

$3^{k+1} < (k+1)!$

5.1/16

$P(0): 0=0$

$P(k) \to P(k+1)$

$$k(k+1)(k+2) + k+1(k+2)(k+3) = \frac{k(k+1)(k+2)(k+3)}{4} + k+1(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3) + 4[(k+1)(k+2)(k+3)]}{4}$$

$$= \frac{(k+1)(k+2)(k+3)[k+4]}{4}$$

5.1/34 $P(n): 6 | n^3 - n$

$\forall n \geq 0\ P(n)$

$P(0): 0$

$P(k) \to P(k+1)$

$6 | k^3 - k$

$6 (k+1)^3 - (k+1)$

$6 | k^3 + 3k^2 + 3k + \cancel{1} - k - \cancel{1}$

$\frac{k^3 + 3k^2 + 3k - k}{6}$

$\frac{k^3 - k}{6} + \frac{3k^2 + 3k}{6}$

$\frac{k^3 - k}{6} + \frac{3k(k+1)}{6} \Rightarrow 6 | k^3 - k + 3k^2 + 3k$

5.1/42 $A_1, A_2 \ldots A_n \quad B$

$$(A_1 - B) \cap (A_2 - B) \ldots (A_n - B) = (A_1 \cap \ldots \cap A_n) - B$$

$$P(1) = A_1 - B = A_1 - B$$

$$P(k): (A_1 - B) \cap (A_1 - B) \cap \ldots \cap (A_k - B) = (A_1 \cap A_2 \cap \ldots A_k) - B$$

$$P(k+1): \underbrace{(A_1 - B) \cap (A_2 - B) \ldots \cap (A_k - B)} \cap (A_{k+1} - B) \stackrel{?}{=} (A_1 \cap A_2 \ldots \cap A_k \cap A_{k+1}) - B$$

$$\boxed{(A_1 \cap A_2 \cap \ldots \cap A_k) - B} \cap (A_{k+1} - B)$$

$$(A_1 \cap A_2 \cap \ldots \cap A_k) \cap \overline{B} \cap A_{k+1} \cap \overline{B}$$

$$(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) \cap \overline{B}$$

$$(A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) - B$$