

Recitation

Dec. 2 2014

Chapter 9

9.5

② $\{ (a,b) \mid a \& b \text{ are the same age} \}$

ref (a,a)

sym $(a,b) (b,a)$

trans $(a,b) (b,c) \rightarrow (a,c)$

$\{ (a,b) \mid a, b \text{ have same parents} \}$

(a,a)

$(a,b) (b,a)$

$(a,b) (b,c) \rightarrow (a,c)$

$\{ (a,b) \mid a, b \text{ share common parents} \}$

* not transitive

ref: (a,a)

Sym: $(a,b) (b,a)$

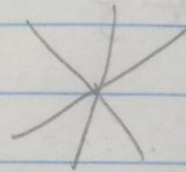
$\{ (a,b) \mid a, b \text{ have met} \}$

* not transitive

9.5-4

$\{(a,b) \mid a \text{ \& } b \text{ are in the same partition}\}$

P_1 / P_2
boys / girls



(a,a)
 $(a,b)(b,a)$
 $(a,b)(b,c)$

9.5.8

$\text{Pow}(R) = \{S \mid S \text{ is a subset of } R\}$

$SRT = (S,T) \iff |S| = |T|$

$R = \{(S,T) \mid |S| = |T|\}$

$R = \{(S,T) \mid S \text{ and } T \text{ have the same cardinality}\}$

proof

$$|S| = |S|$$

$$\forall S \in \text{Pow}(R)$$

$$\rightarrow (S, S) \in R$$

$$\rightarrow SRS$$

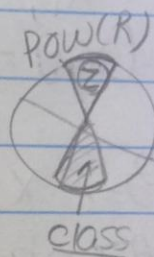
diff.

notations

$$\hookrightarrow (SS) \in R$$

$$\hookrightarrow (S, T) \in R \rightarrow (T, S) \in R$$

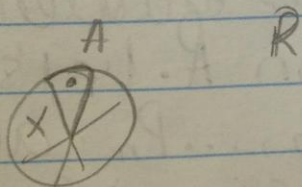
$$\hookrightarrow (S, T), (T, W) \rightarrow (S, W)$$



$$[\{0, 1, 2\}]_R = \{S \mid S \subseteq R \text{ and } |S| = 3\}$$

$$[Z]_R = \{S \mid S \subseteq R \text{ and } |S| = |Z|\}$$

$$\boxed{9.5 - 10} -$$



$$\exists f: A \rightarrow$$

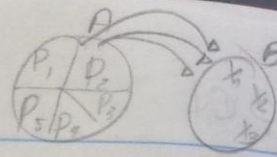
$$(x, y) \in R \leftrightarrow f(x) = f(y)$$

solution:

$$\exists f: A \rightarrow B$$

$$x \rightarrow x_i$$

$$(x, y) \in R \iff f(x) = f(y)$$



$$\rightarrow P_1, P_2, \dots, P_n$$

$$x_1, x_2, \dots, x_n$$

$$(x, y) \in R \Rightarrow [x] = [y]$$

$$= P_i$$

$$[x_1] = P_1, [x_2] = P_2 \quad f(x) = f(y) = x_i$$

$$B = \{x_1, x_2, \dots, x_n\}$$

for any

$$x \in A, \exists P_i: x \in P_i$$

Here solution:

Since R is equivalent, it makes a partition A . Let's say that P_1, P_2, \dots, P_n are the partitions. I define set

$$B = \{1, 2, \dots, n\}$$

I define function $f: A \rightarrow B$

$$x \rightarrow i$$

$$x \in P_i$$

$(x, y) \in R \rightarrow x$ and y belong to the same partition k

$$\rightarrow f(x)=k \text{ and } f(y)=k \rightarrow f(x)=f(y)$$

$$f(x)=f(y)=k \text{ by def. } \rightarrow x \in P_k \text{ and } y \in P_k$$

$$y \in P_k$$

$$\rightarrow (x, y) \in R$$

Other exercise: 9.5 - No

$$R = \{ (a, b), (c, d) \} \mid ad = bc \}$$

equivalence!

$$① ((a, b), (a, b)) \rightarrow ab = ab$$

$$② ((a, b), (c, d)) \rightarrow ((c, d), (a, b))$$

$$ad = bc \rightarrow cb = ad$$

$$③ ((a, b), (c, d)) \wedge ((c, d), (e, f)) \rightarrow ((a, b), (e, f))$$

$$ad = bc$$

$$cf = de$$

$$af = be$$

$$\frac{af}{e} = bc$$

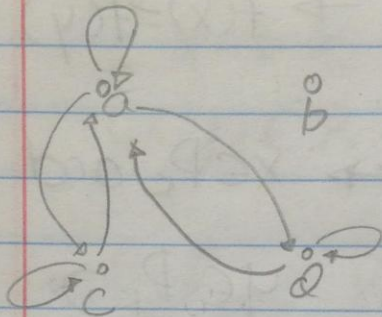
$$d = \frac{cf}{e}$$

$$af = be$$

another way: $\{ (a, b), (c, d) \} \mid ad = bc$
 to help: $\frac{a}{b} = \frac{c}{d}$

q.5

21-23



$$S = \{a, b, c, d\}$$

$$R = \{(a, d), (d, a), (b, b), \dots\}$$

$(a, c) \underbrace{(a, d)} \rightarrow (a, d)$ NO! NO RELATION

$(c, d) \rightarrow$ would make it a relation

q.5-36 $[4]_m$

$$m = 2$$

Solution:

$$[4]_2 = \{x \mid x = 2k\}$$

- $1 \bmod 2 = 1$ "since es ewnda a 1" even
- $2 \bmod 2 = 0$
- $3 \bmod 2 = 1$
- $4 \bmod 2 = 0$

→ if $m=6$

$$[4]_6 = \{x \mid x \bmod 6 = 4\}$$

$$1 \bmod 6$$

$$2 \bmod 6$$

$$3 \bmod 6$$

⋮

$$4 \bmod 6$$

$$\boxed{9.5 - 42}$$

$$\{-3, -2, -1, 0, 1, 2, 3\}$$

- yes. a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$
No, BC "0" is repeated b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$
yes c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1, 0\}$
No, 2 is missing d) $\{-3, 1\}, \{2, 3\}, \{0, -1\}$
yes, has no intersection e) $\{1\}, \{2\}, \{3, -1\}, \{0, -2, -3\}$

$4 \bmod 2 = 0$
 $b) = 3?$

$$[3] = \{3 + 4c \mid c \in \mathbb{Z}\}$$

$$= \{ \dots, -1, 3, 7, \dots \}$$

Q2 Determine the # of diff. equivalence relations on a set with four elements by listing them.

Let set be $\{1, 2, 3, 4\}$

1 partition $\{ \{1, 2, 3, 4\} \}$

2 partitions $\{ \{1\}, \{2, 3, 4\} \}, \{ \{1, 3\}, \{2, 4\} \}, \{ \{1, 2\}, \{3, 4\} \}, \dots$

3 partitions $\{ \{1, 2\}, \{3, 4\} \}, \{ \{1, 4\}, \{2, 3\} \}, \{ \{1, 3\}, \{2, 4\} \}$

1 partition of all separate $\rightarrow \{ \{1\}, \{2\}, \{3\}, \{4\} \}$

0 parti of $(1, 2)(3, 4), (1, 3)(2, 4), (1, 4)(3, 2), (2, 4)(3, 1), (2, 3)(1, 4), (2, 1)(4, 3)$
 $= 5$ in total