Assignment 8 - chapter 6

Sec 6.1

11 - $\bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\ \bigcirc$ 2^8

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

12 - String with length 1 $\bigcirc$ $2 = 2$

" " " 2 $\bigcirc\bigcirc$ $2 \times 2 = 2^2$

" " " 3 $\bigcirc\bigcirc\bigcirc$ $2 \times 2 \times 2 = 2^3$

" " " 4 $= 2^4$

" " " 5   ⋮ $= 2^5$

" " " 6 $\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc$ $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

$$\sum_{i=1}^{6} 2^i = 2^7 - 2 = 126$$

14 - extention of excersise 11 : except two bits at The end we have $n-2$ bits each with 2 Possibility for 0 or 1 Then 2^{n-2}

16 - $\bigcirc\ \bigcirc\ \bigcirc\ \bigcirc$

$x \quad 25 \times 25 \times 25 = 25^3$ with x in The first position

Since x can take 4 positions, and for Each There are 25^3 Possibilities The total would be

$$4 \times 25^3$$

we assumed The strings are of length 4 and There is exactly one x in each string

Sec 6.1 - Continue

24 -

a- $9 \mid 9999$ and $9999 = 1111 \times 9$

so we have 1111 numbers from 1 to 9999

which are divisible by 9.

(namely $1 \times 9, 2 \times 9, 3 \times 9, \dots, 1111 \times 9$)

but some of them are less than 1000 and

we have to exclude.

$1000/9 = 111.\bar{1}$ so 111×9 is the

biggest one we have to exclude. So we have

111 number to exclude namely $1 \times 9, 2 \times 9, \dots, 111 \times 9$

$1111 - 111 = 1000$. The solution ✓

b- $9999 - 1000 + 1 = 9000$ all the numbers we have

Since 9000 is even half of them are even

$9000/2 = 4500$

Sec 6.1 Continue

24) c - O O O O

$9 \times 9 \times 8 \times 7$

Note: The First digit cannot be zero.

d- we better Count Those which are divisible by 3 Then substract from 9000 (The total number of numbers). like part a

$$\left.\begin{array}{l} 9999 = 3 \times 3333 \\ \lfloor 1000/3 \rfloor = 333 \end{array}\right\} \rightarrow 3333 - 333 = 3000$$

$9000 - 3000 = 6000$

e- we have to Count Those which are divisible by 5 and Then Those which are divisible by 7 Then Since The problem asks for "OR" we have to find The union. But Those numbers which are divisible by both 5 & 7 are Count twice if we just add up two numbers. Therefore we have to substract Them

$A = \{x \mid 1000 \leq x \leq 999 \text{ and } 5/x\}$

$B = \{x \mid 1000 \leq x \leq 999 \text{ and } 7/x\}$

Solution $= |A \cup B|$

$= |A| + |B| - |A \cap B|$

Section 6.1 Continue

e- Continued:

(24) $|A| = 1800$:

$\lfloor 9999/5 \rfloor = 1999$

$\lfloor 1000/5 \rfloor = 200$ → but 1000 is included!

$1999 - 200 + 1 = 1800$

$|B| = 1006$:

$\left.\begin{array}{l}\lfloor 9999/7 \rfloor = 1428 \\ \lfloor 1000/7 \rfloor = 142\end{array}\right\} \to 1428 - 142 = 1286$

$|A \cup B| = 1800 + 1286 - |A \cap B| = 3086 - |A \cap B|$

$A \cap B = \{x \mid 1000 < x \leq 9999, 5 \mid x \text{ and } 7 \mid 5\}$

$= \{x \mid 1000 < x \leq 999, 35 \mid x\}$

$|A \cap B| = 257$:

$\left.\begin{array}{l}\lfloor 9999/35 \rfloor = 285 \\ \lfloor 1000/35 \rfloor = 28\end{array}\right\} \to 285 - 28 = 257$

$|A \cup B| = 3086 - 257 = 2829$

Sec 6.1 Continue

24. f- This question asks for $\bar{A} \cup \bar{B}$ with A and B defined in part e

$\bar{A} \cup \bar{B} = \overline{A \cap B}$

$|\overline{A \cap B}| = |U| - |A \cap B|$

$|U| = 9999 - 1000 + 1 = 9000$ all numbers we have

$|A \cap B| = 257$ from part e

$|\bar{A} \cup \bar{B}| = 9000 - 257 = 8743$

g- This asks for $A \cap \bar{B}$

$|A \cap \bar{B}| = |A| - |A \cap B| = 1800 - 257 = 1543$

h- $|A \cap B| = 257$

26- a. First let's count all possible ~~digits~~ strings with no restriction. Then exclude those which have a digit twice. $\underset{10 \times 10 \times 10 \times 10}{O\,O\,O\,O} = 10^4$ is all strings

for a digit x $(x = 0, 1, \ldots, 9)$ we have $\binom{4}{2} = 6$ ways

See 6.1 Continue

to choose 2 position in The string to place x

for each of These 6 ways The other two positions

are free to take any digits among digits $y \neq x$

For example for $x=0$ we have

$0\,0\,\underset{9\times 9}{\bigcirc\,\bigcirc} = 81$ one way

having 6 ways we have 81×6 strings which

have 2 zeros. we have same situation for each

Possibility for x. since x can be $0, 1, \dots,$ or 9

(10 possibilityes) we have $\underbrace{81\times 6\times 10}$ strings

with a digit That repeats twice " 4860

$10^4 - 4860 = 5140$

we have excluded strings like 0011 twice so we have to

add Them up again They are $\underbrace{\binom{10}{2}\binom{4}{2}}_{=270}$ strings

$5140 + 270 = \boxed{5410}$

Sec 6.1 Continue

let's do it easier:

allowed strings are Those with

1. all digits The same
2. 3 of Them The same
3. non of Them The sam

we have to add Them up.

count first group: all could be 0s or 1s or ... 9s

10 strings

Count second group: 4 possibilities for The position That Contains different digit. 10 possibility for The repeated digit and 9 possibili. for that content of The single differt Position = $9 \times 4 \times 10 = 360$

Count Third group: 0000

$10 \times 9 \times 8 \times 7 = 5040$

$5040 + 360 + 10 = \boxed{5410}$

See 6.1 continue

26-b O O O O

$10\times10\times10\times5 = 5000$

c- choosing 3 positions for 9s can be done in

4 ways $\binom{4}{3}$ or $\binom{4}{1}$

For each way we have 9 digits that can put in the fourth place $(0,1,\dots,8)$

$4\times9 = 36$

36. 2^n

all numbers $1,2,\dots,n$ must be assigned to something. one option at a time. And 2 possible options. So $\overset{1}{O}\overset{2}{O}\overset{3}{O}\cdots\overset{n}{O}$

$2\times2\times2\times\cdots\times2$

46. O O O O O O

$10\times9\times8\times7\times6\times5$ all possibilities without restriction

a) O O O O O

$9\times8\times7\times6\times5$ for other people and 6 places for

~~b)~~ bride so $9\times8\times7\times6\times5\times6$

another way $\binom{9}{5}6! = \frac{9\times8\times7\times6\times\cancel{5!}}{\cancel{5!}\times4!}\times6!$

$= 9\times8\times7\times6\times\frac{6\times5\times\cancel{4!}}{\cancel{4!}} = 9\times8\times7\times6\times5\times6$

See 6.1 Contin—

46-b $\binom{8}{4} \cdot 6! = 8 \times 7 \times 6 \times 5 \times 6 \times 5$

4 ways to choose other People among 10-2 People

having 6 People in The Pisture 6! ways to arrenge Then

c. decide one bride or groom First. let's say

bride is in, and groom not.

having The bride in there are 10 - 2 left to

be in. (groom must not be There)

$\binom{8}{5} \cdot 6!$

same ~~amount~~ number of possibilities with groom

$\binom{8}{5} 6!$ Pictures with only groom in

result $= 2 \cdot \binom{8}{5} 6!$

See 6.1 Contin

62. There are n positive integers not exceeding n $(x \leq n)$ we have to exclude those which are not desired. (Those which has p or q in their prime factorization)

here is inclusion-exclusion principal:

$A = \{x \mid x \leq n,$ have p in factorization$\}$

$B = \{x \mid x \leq n,$ have q in factorization$\}$

not desired numbers' $= A \cup B$

$|A \cup B| = |A| + |B| - |A \cap B|$

$A = \{p, 2p, 3p \cdots qp\}$
$B = \{q, 2q, \cdots \quad pq\}$ $\longrightarrow A \cap B = \{pq\}$

$|A| = q \qquad |B| = p \qquad |A \cap B| = 1$

$|A \cup B| = p + q - 1$

as we said we have n numbers and we exclude

"pq"

$q + q - 1$

$pq - (p + q - 1)$

See 6.1 Cont...

74. a. The maximum Value is when all bits are 1s having 16 bits The value in binary is $\underbrace{111\cdots1}_{16\ 1s}$ which is $2^0+2^1+\cdots+2^{15}=2^{16}-1$ in decimal.

b. again The value is max when all bits are 1s, having 4 bits The value in binary is 1111 which is $2^4-1=15$ in decimal.

In octets, 15 means 15 blocks of 8 bits so we have 15×8 bits length for header.

c. Maximum total value $=2^{16}-1$

in octets, max total length of datagram $=8(2^{16}-1)$ bits

if The header is min Then The data would be max

becaus total = header + data

min header is 20×8 bits so max data is

max data = total − min header

max data $=8(2^{16}-1)-8(20)$

$8(2^{16}-1)-8(20)$

d. having $8(2^{16}-1)-8(20)$ bits we have 2 different strings.

$\underbrace{000\cdots0}_{2\times2\times2\times\cdots\times2}\rightarrow 8(2^{16}-1)-8(20)$ bits

Sec 6.2

16. This question is valid only because The given set can be dividet to pairs of numbers which add up to 16. These pairs are: (1,15), (3,13), (5,11), (7,9). having This fact if we choose 5 numbers and assign Them to Their respective pair since we have only 4 pairs by pigeonhole principal two numbers are assigned to one pair That means Their sum is 16. so 5 numbers or more selection guarantees This.

18. a. lets P: "at least 5 male" and q: "at least 5 female".
to prove $P \vee q \equiv T$ we prove an equivalent Proposition which is $\neg P \rightarrow q$ is True. It is not required to prove $q \rightarrow \neg P$ too. (WLG). $\neg P$: "There are maximum 4 males". having exactly 4 males ~~if we assign women to men~~ we have 5 females since male + femal = 9. having less Than 4 males we have more Than 5 femals.

b. maximum 2 femal $\rightarrow$ at least 7 males

2 female $\rightarrow$ 9-2 male
1 female $\rightarrow$ 9-1 male
0 female $\rightarrow$ 9 male

6.2 Continue

32. lets put wage earners in categories below

Cat 1 : Those with 1 penny wage
Cat 2 : 〃 〃 2 〃 〃
⋮
Cat 99,999,999 : 〃 〃 100,000,000-1 penny wage

having more wage earneres than categories, we would have at least 2 people ~~with~~ in the same category by pegionhole principle.

36. lets put computers in categories below

Cat 1 – Those connected directly to 1 other computer
Cat 2 – 〃 〃 〃 〃 2 〃 computers
⋮
Cat 5 – 〃 〃 〃 〃 5 〃 〃

having 6 computer and 5 categories, we would have at least 2 computers in the same category by pegionhole principle.

Sec 6.3

6.3 _10. O O O O O O

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$

14. $\binom{100}{2} = \frac{100!}{98!\,2!} = \frac{\cancel{100}^{50} \times 99 \times \cancel{98!}}{\cancel{98!}\ \cancel{2!}} = 4950$

18. a. O O O O O O O O

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

b. $\binom{8}{3} \rightarrow$ heads to be placed

Since the rest must not be head for them there ~~is~~ only one option (tail)

$$\frac{8!}{3!\,5!} = \frac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{3!} \times \cancel{5!}} = 56$$

c. at least three heads means

total minus (exactly 2 heads + exactly 1 head + no head)

no head $\rightarrow$ one case $\binom{8}{0} = 1$

1 head $\rightarrow \binom{8}{1} = 8$

2 heads $\rightarrow \binom{8}{2} = \frac{8!}{2!\,6!} = \frac{\cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{2} \times \cancel{6!}} = 28$

$28 + 8 + 1 = 37$

total $\rightarrow 2^8 - 37 = 256 - 37 = 219$

6.3 Continu

~~C~~ontinued (18.d)

This equals to exactly 4 hearts $\binom{8}{4}$

22. a. Permutation of ~~6~~ 7 Things:

A, B, C, ED, F, G, H

7!

b. A, B, CDE, F, G, H $\longrightarrow$ 6!

c. BA, C, D, E, FGH $\longrightarrow$ 5!

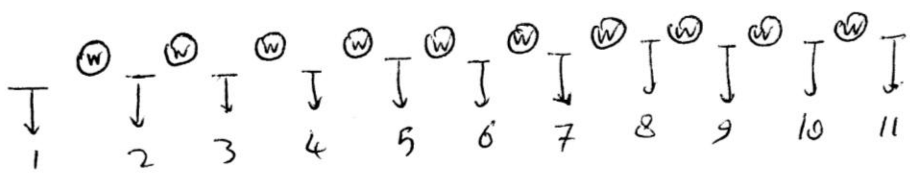
d. AB, DE, GH, C, F $\longrightarrow$ 5!

e. CABED, F, G, H $\longrightarrow$ 4!

f. BCA and ABF cannot be in a permutation together

answer = 0

24.



Since men cannot be next to each other The have to stand in The 11 positions specified by numbers, with $\binom{11}{6}$ ways we choose Their positions, for each There are 6! arrangement. for each state of men we have

6.3 Conti...

26 Continued

we have 10! arrangement of women so we have

$10! \times 6! \times \binom{11}{8}$ in total.

28. we have to choose 17 position to place The True statements. so $\binom{40}{17}$

34. 10 m
15 w

Case 1. 6 w 0 m $\longrightarrow \binom{15}{6}$ just choose 6 w

Case 2. 5 w 1 m $\longrightarrow \binom{10}{1} \cdot \binom{15}{5}$

Case 3. 4 w 2 m $\longrightarrow \binom{10}{2} \cdot \binom{15}{4}$

in Case 2 for any possibility for choosing a man among 10 men we have $\binom{15}{5}$ possibility for choosing women of The Committy So we have to multiply Them. same for Case 3.

Sec 6.3 Continue

42. distinct circular arrangements of p object are $(p-1)!$ ways, because :

First Count all (not distinct) arrangements. This is equal to permutations of p objects which is $p!$

For each arrangement among This $p!$ arrangements Ther are p arrangement which are not distinct, because if we mark one of objects in The arrangement and turn The arrangement such That The marked object sits in its adjasent seat without changing The order (and same for all objects in The arrangement) we don't get a new distinct arrangement. Turning This arranget p times such That The marked object sits in all seats gives p equal arrangement. using The division rule we have $\frac{(p)!}{p} = (p-1)!$ distinct ~~circ~~ circular arrangement.

Now we want to choose r People among n People Then sit Them in a circular table. For choosing we have $\binom{n}{r}$ ways for each way $(r-1)!$ arrangement so: $\binom{n}{r}(r-1)!$

Section $\boxed{6.4}$ 8, 14, 20, 22, 24, 28

8 What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$

$$\binom{17}{9}(3x)^8(2y)^9$$

$$\frac{17!}{8!\,9!}(3)^8(2)^9x^8y^9 = (24{,}310)(6561)(512)$$
$$= 81662929920\,x^8y^9$$

14 ? Show that if "n" is a positive integer, then

$$1=\binom{n}{0}<\binom{n}{1}<\dots<\binom{n}{\lfloor n/2\rfloor}=\binom{n}{\lceil \frac{n}{2}\rceil}>\dots>\binom{n}{n-1}>\binom{n}{n}=1$$

① let's prove the ends

$$1=\binom{n}{0}=\frac{n!}{0!\,n!}=\binom{n}{n}=\frac{n!}{0!\,n!} \Rightarrow \underline{\text{True}} \quad 1=\binom{1}{0}<\binom{n}{1}=n$$

② $$\binom{n}{m+1}=\frac{n!}{(n-m-1)!(m+1)!}=\frac{(n-m)}{(m+1)}\,\frac{n!}{m!(n-m)!}=\frac{n-m}{m+1}\binom{n}{m}$$

bigger than the one b4

③
$\binom{n}{m+1}>\binom{n}{m} \overset{\text{iff}}{\Longleftrightarrow} \frac{n-m}{m+1}>1$. This happens when $n-m>m+1$, or when $m<\frac{n}{2}-\frac{1}{2}$. If n is even, this is equivalent to saying $m<\frac{n}{2}$; for n odd, it means $m<\lfloor \frac{n}{2}\rfloor$. Similarly, $\binom{n}{m+1}<\binom{n}{m}$ for $m>\frac{n}{2}-\frac{1}{2}$; for n even this is for $m\geq\frac{n}{2}$; for n odd it means for $m\geq\lceil\frac{n}{2}\rceil$. Finally, $\binom{n}{m}=\binom{n}{m+1}$ only when $m=\frac{n}{2}-12$, which can only happen when n is odd; in this case $\binom{n}{\lfloor\frac{n}{2}\rfloor}=\binom{n}{\lceil\frac{n}{2}\rceil}$.

(20) Suppose that k and n are integers with $1 \le k \le n$. Prove the hexagon identity

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$$

which relates terms in pascal's triangle that form a hexagon.

$$\left(\frac{(n-1)!}{\underbrace{(n-1-k+1)!}_{(n-k)!}(k-1)!}\right)\left(\frac{n!}{(k+1)!(n-k-1)!}\right)\left(\frac{(n+1)!}{(n+1-k)!(k)!}\right)$$

$$= \frac{\overset{①}{(n-1)!}\,\overset{②}{(n!)}\,\overset{③}{(n+1)!}}{\underset{③}{(n-k)!}\,\underset{②}{(k-1)!}\,\underset{③}{(k+1)!}\,\underset{①}{(n-k-1)!}\,\underset{②}{(n-k+1)!}\,\underset{①}{(k)!}}$$

$$= \underset{①}{\binom{n-1}{k}}\underset{②}{\binom{n}{k-1}}\underset{③}{\binom{n+1}{k+1}}$$ equivalent to right side.

22). ? Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are nonnegative integers with $r \leq n$ and $k \leq r$,

a) using a combinatorial argument
b) using an argument based on the formula for the # of r-combinations of a set with n elements.

a) $\binom{n}{r}\binom{r}{k}$ · out of n faculties, r faculties will be selected
· now, out of r faculties, k faculties will be selected.

$\binom{n}{k}\binom{n-k}{r-k}$ · select k senate members.
· select r faculties where the k are members.

$\therefore \binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

left side

b) $\binom{n}{r}\binom{r}{k} \rightarrow \left(\frac{n!}{r!\,(n-r)!}\right)\left(\frac{r!}{(r-k)!\,k!}\right)$

$\rightarrow \frac{n!}{(n-r)!\,(r-k)!\,(k)!}$

equivalent

right side:

$\binom{n}{k}\binom{n-k}{r-k} = \left(\frac{n!}{\cancel{(n-k)!}\,k!}\right)\left(\frac{\cancel{(n-k)!}}{(n-r)!\,(r-k)!}\right) = \frac{n!}{k!\,(n-r)!\,(r-k)!}$

(24) Show that if p is a prime and k is an integer such that $1 \leq k \leq p-1$, then p divides $\binom{p}{k}$.

Property: "if $\gcd(a,b)=1$, and $a \mid bc$ then $a \mid c$" ... (1)

$$\binom{p}{k} = \frac{p(p-1)(p-2)\dots(p-k+1)}{k!}$$

$\gcd(p,k)=1$ for $1 \leq k \leq p-1$

$\Rightarrow k!\binom{p}{k} = p(p-1)(p-2)\dots(p-k+1)$

$p \mid p(p-1)(p-2)\dots(p-k+1)$ is known..

Consequently, $p \mid k!\binom{p}{k}$ ... (2)

Since k is a positive integer such that $k<p$, we have $\gcd(p,k!)=1$

$\Rightarrow$ by (1) and (2) the only possibility is $p \mid \binom{p}{k}$

(28) Show that if n is a positive integer, then
$\binom{2n}{2} = 2\binom{n}{2} + n^2$

a) using a combinatorial argument
b) using pascal's identity

a) consider a set of n men and n women.
Now $\binom{2n}{2}$ is the # of ways of choosing 2 persons from 2n persons.

possibilities:
① 2 from n men
② 2 from n women
③ 1 from men and 1 from women

$\Rightarrow \binom{n}{2}\binom{n}{0} + \binom{n}{1}\binom{n}{1} + \binom{n}{0}\binom{n}{2}$

$\Rightarrow \binom{n}{2} + \binom{n}{1}\binom{n}{1} + \binom{n}{2}$

$\Rightarrow 2\binom{n}{2} + (n)(n)$

$= 2\binom{n}{2} + n^2$

b) $2\binom{n}{2} + n^2$

$= \cancel{2}\left(\frac{n!}{(n-2)!\,\cancel{2!}}\right) + n^2$

$= (n)(n-1) + n^2$

$= 2n^2 - n$

$\binom{2n}{2} \Rightarrow \frac{2n!}{(2n-2)!\,2!}$

$= \frac{(2n)(2n-1)}{2}$

$= \frac{4n^2}{2} - \frac{2n}{2}$

$= 2n^2 - n$

equivalent.

Section 6.5 10, 18, 28, 32, 34 (dos dudas)

10.

A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose?

a) a dozen croissants

$n=6$
$r=12$

$\to C(n+r-1, r) = C(n+r-1, n-1)$

$C(17,12) = C(17,5) = \frac{17\times16\times15\times14\times13}{5\times4\times3\times2\times1} = \frac{742560}{120} = 618$ ways

b) three dozen croissants? = 36 croissants

$r=36$ $\to$ same
$n=6$ process

c) two dozen croissants with at least two of each kind?

$n=6$
$r=12$ $\to$ escogemos 2 de cada sabor, lo que equivale a 12, los otros 12 la buscamos usando la fórmula de unordered selections.

$C(17,12) = \boxed{6188}$

d) two dozen croissants with no more than two broccoli croissants?

no broco $\begin{cases} n=5 \\ r=24 \end{cases}$ $C(28,24) = 20,475$ +

1 broco $\begin{cases} n=5 \\ r=23 \end{cases}$ $C(27,23) = 17,550$ +

2 broco $\begin{cases} n=5 \\ r=22 \end{cases}$ $C(26,22) = 14,950$

$\Bigg\}$ = 52,975

e) two dozen croissants with at least five chocolate croissants and at least three almond croissants?

5 + 3 = 8 donuts already selected.

→ then...: 24 − 8 = 16 = r

n = 6

→ C(21, 16) ⇒ 20,349

f) two dozen croissants with at least one plain croissant, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissants?

• 1 + 2 + 3 + 1 + 2 = 9 croissants chosen

• 24 − 9 = 15

• we need to choose the broccoli croissants.

→ if none:
n = 5
r = 15
C(19, 15) = 3,876

→ if one
n = 5
r = 14
C(18, 14) = 3,060

→ if two
n = 5
r = 13
C(17, 13) = 2,380

→ if three
n = 5
r = 12
C(16, 12) = 1820

Now, adding them up.
total = 11,136 ways.

(18) How many strings of 20-decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s, and three 9s.

Because so many characters are indistinguishable, we have overcounted badly. We can divide out by all the ways that we can arrange identical characters: $\frac{20!}{2!4!3!1!2!3!2!3!} = 5.8663725120000$

(28) Show that there are $C(n+r-q_1-q_2-\cdots-q_r-1, n-q_1-q_2-\cdots-q_r)$ different unordered selections of n objects of r different types that include at least q_1 objects of type one, q_2 objects of type two, ..., and q_r objects of type r.

among the n indistinguishable objects in which there are r diff. types of objects. Here at least q_1 objects of 1st type, q_2 objects of 2nd type, ..., q_r objects of r^{th} type are present.
They can be distributed in.

$$C(n, q_1) \cdot C(n-q_1, q_2) \cdot C(n-q_1-q_2, q_3) \ldots$$

$$C(n-(q_1+q_2+\ldots+q_{r-1}), q_r) = C(n+r-(q_1+q_2+\ldots+q_r)-1, n-q_1-q_2-\ldots-q_r)$$

(32) How many difl. strings can be made from the letters in AARDVARK, using all the letters, if all three A's must be consecutive?

AAA, R, D, V, R, K

1 2 3 4 5 6

$n=6$

$\frac{6!}{2!}$ ← que ya están $=360$

(34) How many strings with 5 or more characters can be formed from the letters in SEERESS?

• Five letter string: $\frac{5!}{3!2!} = 10$ (3 S, two E)

↳ three S, one R, one E = $\frac{5!}{3!1!1!} = 20$

↳ three E, two S = $\frac{5!}{3!2!} = 10$

↳ three E, one S, one R = $\frac{5!}{3!1!1!} = 20$

↳ two S, two E, one R $= \frac{5!}{2!2!1!} = 30$

• Six letters

three S, three E $\frac{6!}{3!3!} = 20$

↳ one R, two S, three E $\frac{6!}{1!2!3!} = 60$

↳ one R, two E, three S $\frac{6!}{1!2!3!} = 60$

• seven letters

$\frac{7!}{3!3!1!} = 140$

total: $10+20+10+20+20+60+60+30+140 = 370$